Post graduate Exam (Basic Engineering Sciences)
Branch: Engineering Mathematics (Master 500)

Menofia University **Faculty of Engineering** **Academic Year: 2014-2015** **Department: Basic Eng. Sci.**		**Subject: Introduction in Mathematical Physics** **Time Allowed: 3 hours** **Date: 3 / 6 / 2015** **Max Marks: 100**

Answer all the following questions:

Q.1 **(A) Define: gamma function, beta function, error function, the Dirac delta function.**

(B) Prove that: $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.

(C) Expressed in terms of integration following gamma functions

$$I(\lambda)=\int_0^{\infty} e^{-\lambda x^3}dx, \lambda \succ 0.$$

(D) Evaluate the following integrals:

⊕ $\int_0^1 (x\ln x)^3 dx$ ⊕ $\int_0^2 \frac{x^2 dx}{\sqrt{2-x}}$

[Q.1 (50 mark)]

Q.2 **(A) Compare between the error function and the complementary error function.**

(B) What are the properties of Dirac delta function?

(C) Solve the following differential equation for Legendre:

$$(1-x^2)y''-2xy'+\ell(\ell+1)y=0$$

(D) Find the solution to the heat equation:

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$$

$$0<x<h, \quad t>0$$

$$u(0,t)=u_x(h,t)=0, \quad u(x,0)=f(x);$$

where k is a positive real number.

(E) Solve the following initial and boundary value problem:

$$u_t=ku_{xx}+s(x,t)$$

$$0<x<h, \quad t>0$$

$$u(0,t)=\alpha(t), \quad u_x(h,t)=\beta(t), \quad u(x,0)=f(x)$$

where k is a positive real number.

[Q.2 (50 mark)]

With my best wishes ***Dr. M.A. El-Shorbagy***